The required problems are Exercises 4.1, 4.2, and 4.5. Please let me know if any of the problems are unclear or have typos.

Exercise 4.1. Suppose that $W \subset \mathbb{R}^n$ is a linear subspace. Define the *orthogonal complement* $W^\perp = \{u \in \mathbb{R}^n \mid \forall w \in W, u \cdot w = 0\}$. Prove that $W^\perp$ is also a linear subspace. Show that $\mathbb{R}^n$ has an orthonormal basis $\{f_i\}$ so that $W = \langle f_1, \ldots, f_k \rangle$ and $W^\perp = \langle f_{k+1}, \ldots, f_n \rangle$. Deduce $\dim(W) + \dim(W^\perp) = n$.

Exercise 4.2. Suppose that $T(x) = Ax + b$ is an isometry of $\mathbb{E}^2$, where A is a non-trivial rotation. Prove that T has a *fixed point*: that is, there is a point $p \in \mathbb{E}^2$ so that $T(p) = p$. (This is a part of Exercise 1.8 in the book.)

Exercise 4.3. Theorem 2.6 states that any isometry $T \in \mathrm{Isom}(\mathbb{E}^n)$ can be realized as the composition of at most $n + 1$ reflections. Below is a sketch of a proof. Look up any unfamiliar terms and then fill in the details.

Theorem 1.11 implies that any $B \in O(n)$ can be realized as the composition of at most n reflections. Now, suppose $T(x) = Ax + b$. Then there is a reflection R so that $R \circ T(0) = 0$. Let $B = R \circ T$. Since $B \in O(n)$, and since reflections are involutions, we are done.

Exercise 4.4. [Hard] Show that Theorem 2.6 is *sharp*: the inequality cannot be improved. Do this by finding, for each n, an isometry $T \in \mathrm{Isom}(\mathbb{E}^n)$ which cannot be realized as a composition of n or fewer reflections.

Exercise 4.5. By Theorem 1.14 any isometry T of $\mathbb{E}^2$ is either a translation, rotation, reflection, or glide reflection. In each case write T as a composition of at most three reflections and draw the appropriate picture.