

The required problems are Exercises 9.1, 9.2, and 9.3. Please let me know if any of the problems are unclear or have typos.

Exercise 9.1. A metric space (X, d_X) is *homogeneous* if for all $p, q \in X$ there is an isometry $T \in \text{Isom}(X)$ so that $T(p) = q$. Show that \mathbb{S}^2 , \mathbb{E}^2 , and \mathbb{H}^2 are all homogeneous. In each case, discuss the uniqueness of the isometry T .

Exercise 9.2. Show that for any pair $u, v \in \mathbb{L}^3$ of light-like vectors there is a matrix $A \in O^+(1, 2)$ with $Au = v$. Prove or disprove: the matrix A is unique.

Exercise 9.3. Suppose that $\Delta, \Delta' \subset \mathbb{H}^2$ are *ideal triangles*: that is, the three sides of Δ are infinite, pairwise asymptotic lines. Show that there is an isometry $T \in \text{Isom}(\mathbb{H}^2)$ so that $T(\Delta) = \Delta'$. Prove or disprove: the isometry T is unique.

Exercise 9.4. [Medium.] Suppose that $\Delta, \Delta' \subset \mathbb{H}^2$ are triangles with interior angles $\alpha = \alpha'$, $\beta = \beta'$, and $\gamma = \gamma'$. Suppose that the angles α , β , and γ are all distinct and positive. Show that there is an isometry $T \in \text{Isom}(\mathbb{H}^2)$ so that $T(\Delta) = \Delta'$. Prove or disprove: the isometry T is unique.

Exercise 9.5. Exercise 4.4 from Chapter 4 in the book.

Exercise 9.6. Suppose that $T(x) = b + Ax$, where $A \in \text{GL}(n, \mathbb{R})$ and $b \in \mathbb{R}^n$. Show that T sends affine subspaces of \mathbb{A}^n to affine subspaces.