The required problems are Exercises 3.1, 3.3, and 3.5. Please let me know if any of the problems are unclear or have typos.

For the first three problems the paths $f, g, h\colon I \to X$ are loops based at the point $x \in X$. The path $e\colon I \to X$ is the constant loop, also based at x.

Exercise 3.1. Give explicit parameterizations of the loops $p_0 = e * g$ and $p_1 = g * e$. Show, by giving a picture in $I \times I$, a picture in X, and an explicit homotopy, that p_0 and p_1 are homotopic (preserving endpoints). Do the same for g and p_0.

Exercise 3.2. Define $\bar{g}\colon I \to X$ by $\bar{g}(s) = g(1 - s)$. Give an explicit parameterization of the loop $p = g * \bar{g}$. Show, by giving a picture in $I \times I$, a picture in X, and an explicit homotopy, that p and e are homotopic (preserving endpoints). Briefly discuss the corresponding situation for $q = \bar{g} * g$.

Exercise 3.3. Give explicit parameterizations of the loops $p_0 = (f * g) * h$ and $p_1 = f *(g * h)$. Show, by giving a picture in $I \times I$, a picture in X, and an explicit homotopy, that p_0 and p_1 are homotopic (preserving endpoints).

Exercise 3.4.

- Let $X \subset \mathbb{R}^3$ be the union of the coordinate axes. Show that $\mathbb{R}^3 - X$ is homotopy equivalent to a graph. Which graph?
- Let $X \subset \mathbb{R}^4$ be the union of the xy–plane and the zw–plane. Show that $\mathbb{R}^4 - X$ is homotopy equivalent to a surface. Which surface?

Exercise 3.5. Suppose that $p\colon Y \to X$ is a covering map. Recall the definition of $\mathrm{Deck}(p)$ and prove it is a group with respect to function composition.

Exercise 3.6. Show that the map $p\colon \mathbb{R} \to S^1$ defined by $p(t) = \exp(2\pi i t)$ is a covering map. Give an informal proof that $\mathrm{Deck}(p) \cong \mathbb{Z}$. (We will give a careful proof of this later in the course.)