Please let me know if any of the problems are unclear or have typos.

Exercise 11.1. Suppose that X is a topological space. Show that $X \times X$ is not homeomorphic to S^1. [Harder: do the same for S^2.]

We use the following notations for Exercises 11.2 and 11.4. Suppose that X is a path-connected CW complex. Fix $x_0 \in X^0$. Suppose that the universal cover $\widetilde{X}$, the basepoint $\widetilde{x}_0 \in \widetilde{X}$, the covering $p\colon (\widetilde{X}, \widetilde{x}_0) \to (X, x_0)$, the basis $\mathcal{U}$ (for X), and the subsets $U_\gamma \subset \widetilde{X}$ are all as defined in lecture.

Exercise 11.2. Give a careful proof that

$$\widetilde{\mathcal{U}} = \{U_\gamma \mid U \in \mathcal{U}, [\gamma] \in \widetilde{X}, \gamma(1) \in U\}$$

is a basis for a topology on $\widetilde{X}$.

Exercise 11.3. A cover $q\colon Y \to X$ is *normal* if for every $x \in X$ the deck group $\mathrm{Deck}(q)$ acts transitively on $q^{-1}(x)$. Determine which degree-three covers of $S^1 \vee S^1$ are normal.

Exercise 11.4.

- Suppose that γ is a loop in X, based at x_0. Define $\rho_\gamma\colon (\widetilde{X}, \widetilde{x}_0) \to (\widetilde{X}, \widetilde{x}_0)$ by $\rho_\gamma([\alpha]) = [\gamma * \alpha]$. Prove that ρ_γ is well-defined and is a deck transformation.
- Recall that p is the given covering map from $\widetilde{X}$ to X. Define $\rho\colon \pi_1(X, x_0) \to \mathrm{Deck}(p)$ by $\rho([\gamma]) = \rho_\gamma$. Prove that ρ is well-defined and is a group isomorphism.
- Suppose that $q\colon (Y, y_0) \to (X, x_0)$ is a pointed, connected cover. Find a pointed map $r\colon (\widetilde{X}, \widetilde{x}_0) \to (Y, y_0)$ so that r is a pointed covering and so that $p = q \circ r$.

Exercise 11.5. [Hard.] Suppose that X, Y, and Z are topological spaces. Suppose that $q\colon Y \to X$ and $p\colon Z \to X$ are covering maps and suppose that $r\colon Z \to Y$ is a map with $p = q \circ r$. Must r be a covering map? Prove this or give a counterexample. [Hint: see exercise 16, page 80, of Hatcher. Be aware of differences in notation.]