Please let me (Saul) know if any of the problems are unclear or have typos. Please turn in a solution to Exercise 5.3, or to the first two parts of Exercise 5.5, or to Exercise 5.6 by 14:00 on 2017-11-09 in front of the undergraduate office. If you collaborate with other students, please include their names.

**Exercise 5.1.** Define the punctured plane to be $\mathbb{C}^\times = \mathbb{C} - \{0\}$. Show that the map $p\colon \mathbb{C}^\times \to \mathbb{C}^\times$ defined by $p(z) = z^2$ is a covering map. Explain why the squaring map on $\mathbb{C}$ itself is not a covering map.

**Exercise 5.2.** [Exercise 12, page 39, of Hatcher.] Show that for every homomorphism $\phi\colon \pi_1(S^1, 1) \to \pi_1(S^1, 1)$ there is a pointed map $f\colon (S^1, 1) \to (S^1, 1)$ so that $\phi = f_*$. In other words, $f$ induces $\phi$.

**Exercise 5.3.** Suppose that $p\colon \widetilde{X} \to X$ is a covering map, and suppose that $\widetilde{X}$ is path-connected. Show that $\tau \in \mathrm{Deck}(p)$ fixes a point of $\widetilde{X}$ if and only if $\tau = \mathrm{Id}_{\widetilde{X}}$.

**Exercise 5.4.** Suppose that $p\colon \mathbb{R} \to S^1$ is the usual covering map, namely $p(t) = \exp(2\pi i t)$. Give a complete proof that $\mathrm{Deck}(p) \cong \mathbb{Z}$.

**Exercise 5.5.** [Exercise 16, page 39, of Hatcher.] Show that there is no retraction $r\colon X \to A$ in any of the following cases. (Give short justifications of any fundamental group computations.)

- $X = \mathbb{R}^3$ with $A$ any subspace homeomorphic to $S^1$.
- $X = S^1 \times D^2$ with $A$ its boundary torus $S^1 \times S^1$.
- $X = S^1 \times D^2$ and $A$ the circle shown in the figure. [See book.]
- $X = D^2 \vee D^2$ with $A$ its boundary $S^1 \vee S^1$.
- $X$ a disk with two points on its boundary identified and $A$ its boundary $S^1 \vee S^1$.
- $X$ the Möbius band and $A$ its boundary circle.

**Exercise 5.6.** [Hard.] We say that a space $X$ has the *fixed point property* if every map $f\colon X \to X$ has a fixed point. Define the *tripod* to be the set

$$T = \big\{ r \exp(2\pi i k/3) \in \mathbb{C} \mid r \in [0, 1], k \in \{0, 1, 2\} \big\}.$$

So $T$ is a connected graph with three edges and four vertices. Prove the tripod has the fixed point property.