Please let me (Saul) know if any of the problems are unclear or have typos. Please turn a solution to one of Exercise 1.2, Exercise 1.3, or Exercise 1.5 on 2017-01-18 by 14:00. If you collaborate with other students, please include their names.

Exercise 1.1. Show that the map $f\colon [0,1) \to S^1$, given by $f(t) = \exp(2\pi i t) = \cos(2\pi t) + i\sin(2\pi t)$, is a continuous bijection, but is not a homeomorphism.

Exercise 1.2. Arrange the capital letters of the Roman alphabet, ABCDEFGHI-JKLMNOPQRSTUVWXYZ thought of as graphs, into homeomorphism classes. Briefly explain your reasoning. (It will make the problem easier if you use a different font!)

Exercise 1.3. Recall that B^n is the closed unit ball in $\mathbb{R}^n$ while S^n is the unit sphere in $\mathbb{R}^{n+1}$. Show that no two of the interval B^1, the circle S^1, the disk B^2, and two-sphere S^2 are homeomorphic.

Exercise 1.4. As in Exercise 1.2 classify the capital letters of the alphabet into homeomorphism types; this time, we think of the letters as small two-dimensional neighborhoods of the given planar graphs.

Exercise 1.5. Let $A\colon \mathbb{Z}^3 \to \mathbb{Z}^3$ be the homomorphism given by the following matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Compute the kernel, image, and cokernel of A.

Exercise 1.6. [Reading exercise.] The real projective space $\mathbb{RP}^n$ is the space of lines through the origin in $\mathbb{R}^{n+1}$. This can also be described as the quotient of S^n by the antipodal map. The group $\mathrm{SO}(n)$ is the group of orthogonal n-by-n matrices with determinant one.

- Show that $\mathrm{SO}(2)$ is homeomorphic to S^1.
- Show that $\mathrm{SO}(3)$ is homeomorphic to $\mathbb{RP}^3$. It follows that $\pi_1(\mathrm{SO}(3)) \cong \mathbb{Z}/2\mathbb{Z}$. Decribe the generator of $\pi_1(\mathrm{SO}(3))$ directly, and explain why traversing this loop twice gives a homotopically trivial loop.
- [Hard.] Recall that S^3 is a group via its identification with the unit quaternions $U\mathbb{H}$. Describe, in terms of the natural coordinates, the group homomorphism $U\mathbb{H} \to \mathrm{SO}(3)$ that corresponds to the quotient map $S^3 \to \mathbb{RP}^3$.