Please let me (Saul) know if any of the problems are unclear or have typos.

Exercise 10.1. [Medium.] Recall that $H_2(T^2) \cong \mathbb{Z}$. Define the *degree* of a map $f\colon T^2 \to T^2$ to be the degree of the induced homomorphism $f_2\colon H_2(T^2) \to H_2(T^2)$. Recall that $T^2 \cong \mathbb{R}^2/\mathbb{Z}^2$ (the coset space). For any two-by-two integer matrix M define $f_M\colon T^2 \to T^2$ via $f_M([v]) = [M(v)]$.

- Conjecture a relationship between $\deg(f_M)$ and the entries of M; verify your conjecture for several matrices M.
- Prove your conjecture.

Exercise 10.2. Show that S^n, P^n, and T^n — the sphere, projective space, and torus — are manifolds.

Exercise 10.3. Show that S^n and T^n are orientable. Show that P^n is orientable if and only if n is odd.

Exercise 10.4. [Hatcher page 157, problem 27.] Suppose that (X, A) is a pair. Prove that the short exact sequence $0 \to C_n(A) \to C_n(X) \to C_n(X, A) \to 0$ splits. Does this imply $H_n(X) \cong H_n(A) \oplus H_n(X, A)$? Explain.

Exercise 10.5. Recall that S_g is the closed (compact without boundary), connected, orientable surface of genus g. That is, S_g is obtained by attaching g handles to a planar surface with g boundary components. Prove that S_g is homeomorphic to the following CW-complex, having one vertex, $2g$ edges labelled $\{a_i, b_i\}$, and a single 2–cell attached via the path $a_1 b_1 \bar{a}_1 \bar{b}_1 a_2 b_2 \bar{a}_2 \bar{b}_2 \ldots a_g b_g \bar{a}_g \bar{b}_g$.

Exercise 10.6. [Medium. Hatcher page 19, problem 16.] Prove that S^∞ is contractible.

Exercise 10.7. Compute the homology groups of P^∞, of $T^n = \times^n S^1$, and of $S^\ell \times S^m$.

Exercise 10.8. [Hatcher page 157, problems 20–23.] Suppose that X and Y are finite CW–complexes. Prove any one of the following.

- $\chi(X \sqcup Y) = \chi(X) + \chi(Y)$.
- $\chi(X \times Y) = \chi(X) \cdot \chi(Y)$.
- If X is the union of subcomplexes A and B then $\chi(X) = \chi(A) + \chi(B) - \chi(A \cap B)$.
- If $p\colon X \to Y$ is an n–fold covering map then $\chi(X) = n \cdot \chi(Y)$.
- If $p\colon S_h \to S_g$ is an n–fold covering map (of surfaces) then $h = n(g - 1) + 1$. Show that this is the only restriction.