

Please let me (Saul) know if any of the problems are unclear or have typos. Please turn a solution to one of Exercise 2.5, Exercise 2.6, or Exercise 2.7 on 2017-01-25 by 14:00 in front of the undergraduate office. If you collaborate with other students, please include their names.

Exercise 2.1. [Medium. Hatcher page 104 and page 522.] Suppose that X is a topological space, equipped with a Δ -complex structure $\{\sigma_\alpha: \Delta_\alpha \rightarrow X\}_{\alpha \in J}$. Prove that X is Hausdorff.

Exercise 2.2. [Medium. Hatcher page 130 and page 520.] Suppose that X is a topological space, equipped with a Δ -complex structure $\{\sigma_\alpha: \Delta_\alpha \rightarrow X\}_{\alpha \in J}$. Suppose that $K \subset X$ is compact. Prove that K meets only finitely many open simplices $\sigma_\alpha(\Delta_\alpha)$.

Exercise 2.3. Show that every compact, connected, orientable surface without boundary admits a Δ -complex structure.

Exercise 2.4. [Medium.] Suppose that X and Y are equipped with Δ -complex structures. Show that $X \times Y$ admits a Δ -complex structure.

Exercise 2.5. Show that \mathbb{Q} is not isomorphic to a free abelian group.

Exercise 2.6. [Hatcher's extra problems, 2.1.1.] Let X be the circle, equipped with the Δ -complex structure with n vertices and n edges. Compute the simplicial homology of X , directly from the definitions.

Exercise 2.7. List all Δ -complexes that can be made from a single two-simplex. (I believe there are seven.) For each, compute its fundamental group and all of its simplicial homology groups.

Exercise 2.8. [Hard.] Compute the simplicial homology groups of Δ^n , the n -simplex equipped with the natural Δ -complex structure.

Exercise 2.9. [Hard.] Let $C(r)$ be the circle in the plane centered at $(r, 0)$ and with radius r . Define $H = \cup_{n \in \mathbb{Z}_+} C(1/n)$ and endow it with the subspace topology. This is the *Hawaiian earring*. Show that H does not admit a Δ -complex structure.