Please let me (Saul) know if any of the problems are unclear or have typos. Please turn a solution to one of Exercise 5.5, Exercise 5.6, or Exercise 5.8 by 14:00 on 2017-02-15, in front of the undergraduate office. If you collaborate with other students, please include their names.

Exercise 5.1. [Hard.] Suppose that X is a Δ–complex and suppose that A is a subcomplex. Show that (X, A) is a good pair. (Try doing this first when A is a single vertex. See pages 522-523 of Hatcher for discussion.)

Exercise 5.2. Suppose that $A \subset X$ where X is path-connected and A is non-empty. Show that $H_0^s(X, A) = 0$, directly from the definitions.

Exercise 5.3. [Hatcher page 118.] We use $\widetilde{\mathcal{C}}_*^s(X)$ to represent the *reduced* chain complex. That is, set $C_{-1}^s(X) = \mathbb{Z}$ and replace $\partial_0 = 0$ by the *augmentation map* $\epsilon\colon C_0^s(X) \to C_{-1}^s(X)$, where $\epsilon(\sum n_\alpha v_\alpha) = \sum n_\alpha$. If (X, A) is a pair with $A \neq \emptyset$ then show that there is a short exact sequence of chain complexes $0 \to \widetilde{\mathcal{C}}_*^s(A) \xrightarrow{i} \widetilde{\mathcal{C}}_*^s(X) \xrightarrow{q} \mathcal{C}_*^s(X, A) \to 0$. Thus there is an exact triangle of reduced and relative homologies.

Exercise 5.4. [Hatcher page 118.] Suppose that $B \subset A \subset X$; we say (X, A, B) is a *triple*. Show that the inclusion and quotient maps give a short exact sequence of chain complexes $0 \to \mathcal{C}_*(A, B) \xrightarrow{i} \mathcal{C}_*(X, B) \xrightarrow{q} \mathcal{C}_*(X, A) \to 0$. Thus there is an exact triangle of relative homologies.

Exercise 5.5. [Medium. Hatcher page 113, problem 25.] Let $X = [0, 1]$ and set $A = \{0\} \cup \{1/n \mid n \in \mathbb{Z}_+\}$. Show (X, A) is not a good pair. Show $H_1^s(X, A)$ is not isomorphic to $H_1^s(X/A)$.

Exercise 5.6. [Hatcher page 132 and pages 147-148, problem 11.] Suppose that $A \subset X$ is a subset. Let $i\colon A \to X$ be the inclusion map. Suppose that A is a *retract* of X: that is, there is a map $r\colon X \to A$ with $r \circ i = \mathrm{Id}_A$. Show that there is an isomorphism $H_*^s(X) \cong H_*^s(A) \oplus H_*^s(X, A)$.

Exercise 5.7. [Hatcher page 119.] Prove that the two versions of excision are equivalent.

Exercise 5.8. After recalling the necessary definitions from Hatcher's proof of excision, verify the following formulas.

- $\partial b + b\partial = \mathbb{1}$.
- $\partial S = S\partial$ and $\partial T + T\partial = \mathbb{1} - S$.
- $\partial S^m = S^m\partial$ and $\partial D_m + D_m\partial = \mathbb{1} - S^m$.
- $\partial\rho = \rho\partial$ and $\partial D + D\partial = \mathbb{1} - \iota \circ \rho$.