Please let me (Saul) know if any of the problems are unclear or have typos. Please turn a solution to one of Exercise 7.1, Exercise 7.3, or Exercise 7.6 by 14:00 on 2017-03-01, in front of the undergraduate office. If you collaborate with other students, please include their names.

Exercise 7.1. Suppose X is a non-empty finite graph without isolated vertices. Compute the local homology groups $H_*(X, X - x)$ for all $x \in X$.

Exercise 7.2. Define $\mathbb{R}^\infty$ to be the set of sequences of real numbers where all but finitely many terms are zero. Equip $\mathbb{R}^\infty$ with the usual Euclidean distance and define $B^\infty = \{x \in \mathbb{R}^\infty : |x| \leq 1\}$ to be the unit ball. Find a continuous function $h\colon B^\infty \to B^\infty$ without fixed points.

Exercise 7.3.

- Suppose that $\{(X_\alpha, x_\alpha)\}$ is a family of pointed spaces where each (X_α, x_α) is a good pair. Let $X = \sqcup X_\alpha$ and $A = \sqcup\{x_\alpha\}$ be the corresponding disjoint unions. Prove that (X, A) is a good pair.
- Let $W = \vee_{i=0}^{\infty} S^1$ be the countable wedge of circles; let H be the Hawaiian earring. Give a continuous bijective map $f\colon W \to H$.
- Give a two-line proof that W and H are not homeomorphic. This gives another example in the spirit of Exercise 1.1. See also the discussions at Wikipedia, MathOverflow, the maths site at StackExchange, etc.

Exercise 7.4. [Hatcher page 132, problem 15.] Suppose that (X, A) is a pair. Show that the inclusion $i\colon A \to X$ induces an isomorphism $i_n\colon H_n(A) \xrightarrow{\sim} H_n(X)$ for all n if and only if the relative homology $H_n(X, A)$ vanishes for all n.

Exercise 7.5. [Medium. Hatcher page 132, problem 19.] Let X be the subspace of the unit square, $[0, 1]^2$, consisting of the four sides and of all points with rational first coordinate. Compute the homology groups $H_*(X)$.

Exercise 7.6. [Hatcher page 133, problem 29.]

- Compute the singular homology groups of $T^2 = S^1 \times S^1$ and of $X = S^1 \vee S^1 \vee S^2$.
- Prove that T^2 and X are not homotopy equivalent. (This gives one possible solution to Exercise 4.8.)