Please let me (Saul) know if any of the problems are unclear or have typos. Please turn a solution to one of Exercise 8.1, Exercise 8.2, or Exercise 8.5 by 14:00 on 2017-03-08, in front of the undergraduate office. If you collaborate with other students, please include their names.

Exercise 8.1. For each of the pairs (X, A) given in Exercise 6.1 compute the associated long exact sequence of homology. Show, in each case, that A is not a retract of X.

Exercise 8.2. [Hatcher page 132, problem 22, and page 140.] Suppose that X is a Δ–complex. Show the following.

- If X has no $(n+1)$–simplices, then $H_n(X)$ is a free abelian group.
- The number of n–simplices in X is an upper bound for the size of a minimal generating set for $H_n(X)$.

Let $X^k \subset X$ be the k–skeleton.

- The inclusion $i\colon X^k \to X$ induces an isomorphism $i_n\colon H_n(X^k) \cong H_n(X)$ when $k > n$.
- The previous statement may fail when $k = n$.

Exercise 8.3. [Medium. Hatcher page 156, problem 16.] Suppose $X = (\Delta^m)^k$ is the k–skeleton of the m–simplex. Compute the reduced homology groups of X.

Exercise 8.4. Suppose that M is an m–manifold and N is an n–manifold. Show that $M \times N$ is an $(m+n)$–manifold.

Exercise 8.5. Suppose that M is an m–manifold and N is an n–manifold. Show that if M is homeomorphic to N then $m = n$.

Exercise 8.6. Suppose that M is a m–manifold with boundary. Define ∂M to be the subspace of M consisting of points which do not have a neighborhood homeomorphic to $\mathbb{R}^m$. Show that ∂M is an $(m-1)$–manifold (or is empty).

Exercise 8.7. Prove that the real projective space $\mathbb{RP}^n$ is an n–manifold.

Exercise 8.8. Prove that the complex projective space $\mathbb{CP}^n$ is a $2n$–manifold.

Exercise 8.9. Let $\Sigma = \Sigma_g$ be the compact connected oriented surface, without boundary, of genus g. That is, Σ is obtained by attaching g "handles" to a planar surface with g boundary components. Compute $H_*(\Sigma)$.