Please let me know if any of the problems are unclear or have typos.

Exercise 2.1. We denote the connect sum of n copies of M by nM. Verify the homeomorphisms of surfaces.

- $K^2 \cong 2P^2$

- $T^2 \# P^2 \cong 3P^2$.

Exercise 2.2. We denote the (p, q) lens space by $L(p, q)$. Also, $\mathrm{UT}(M)$ is the unit tangent bundle to the manifold M. Verify the following homeomorphisms of elliptic manifolds.

- $L(1, 1) \cong S^3 \cong \mathrm{SU}(2)$ (the special unitary group).

- $L(2, 1) \cong P^3 \cong \mathrm{UT}(S^2) \cong \mathrm{SO}(3)$.

- $L(4, 1) \cong \mathrm{UT}(P^2)$. Also, show that $L(4, 1)$ is a prism manifold.

Exercise 2.3. Suppose that $L = L(p, q)$ and $L' = L(p', q')$ are lens spaces.

- Suppose that $L \cong L'$. Prove that $p = p'$.

- Give necessary and sufficient conditions on q and q' to ensure that $L \cong L'$.

- [Hard.] Give necessary and sufficient conditions on q and q' to ensure that L is homotopy equivalent to L'.