

Please let me know if any of the problems are unclear or have typos.

**Exercise 4.1.** Copy the proof of Alexander's theorem to give a proof of the easy part of the Jordan-Schoenflies theorem in dimension two: Every smooth curve in  $S^2$  divides the sphere into two connected components. The closure of each is a disk.

**Exercise 4.2.** Suppose that  $S \subset S^3$  is a smoothly embedded two-sphere. Let  $h: \mathbb{R}^3 \rightarrow \mathbb{R}$  be the projection onto the third coordinate. Define  $H = h|_S$  and suppose that  $H$  is Morse. Suppose that  $c$  is a critical value of  $H$  and  $\epsilon$  is sufficiently small. Suppose  $C$  is a connected component of  $H^{-1}([c - \epsilon, c + \epsilon])$ . Classify the possible homeomorphism types for  $C$ , including how it embeds into and separates the region  $P = h^{-1}([c - \epsilon, c + \epsilon])$ .

**Exercise 4.3.** Suppose that  $S, S' \subset \mathbb{R}^3$  are smoothly embedded two-spheres. Show that there is an ambient isotopy taking  $S$  to  $S'$ . [Finding a diffeotopy is harder.]

**Exercise 4.4.** [Hard.] Compute the homotopy type of  $\text{Emb}(S^2, S^3)$ , the space of smooth embeddings of the two-sphere into the three-sphere.

**Exercise 4.5.** Suppose that  $T \subset S^3$  is a smoothly embedded two-torus. Show that  $T$  bounds a solid torus on at least one side.