

Please let me know if any of the problems are unclear or have typos.

**Exercise 6.1.** Suppose that  $f: (F, \partial F) \rightarrow (B^3, \partial B)$  is a general position map with two simple branch points, with one double arc, and with one sector (which is an annulus). Enumerate the possibilities and, for each, draw accurate pictures of  $F$ ,  $\Sigma(f)$ , and  $f(F)$ .

**Exercise 6.2.** Suppose that  $f: F \rightarrow S^3$  is a general position immersion. Thus  $f$  has no simple branched points and  $\partial F = \emptyset$ . Suppose that  $f$  has exactly one triple point, three double arcs, and no double curves. Finally, suppose that all sectors of  $f$  are disks. Enumerate as many possibilities as you can, and for each, draw accurate pictures of  $F$ ,  $\Sigma(f)$ , and  $f(F)$ .

**Exercise 6.3.** Show that Dehn's Lemma follows from the Disk Theorem.

**Exercise 6.4.** Suppose that  $K \subset S^3$  is a knot and let  $X_K = S^3 - n(K)$  be its exterior. Show that  $K$  is isotopic to the unknot if and only if  $\pi_1(X_K) \cong \mathbb{Z}$ .

**Exercise 6.5.** Suppose that  $\mathbb{F}_g$  is the free group on  $g$  generators. Suppose that  $M^3$  connected, compact, oriented, irreducible, and has  $\pi_1(M) \cong \mathbb{F}_g$  with  $g > 0$ . Show that  $M$  is a handlebody. [Hint: This is easier to prove with Kneser's Hilfsatz in hand.]