Please let me know if any of the problems are unclear, have typos, or have mistakes.

Exercise 1.1. Suppose that $X = S^2 \times S^4$ and $Y = \mathbb{CP}^3$.

1. Check that X and Y are compact, connected, oriented manifolds without boundary (of the same dimension).
2. Prove that $\pi_1(X)$ and $\pi_1(Y)$ are both trivial.
3. Give a CW–complex structure on each of X and Y.
4. Using this, or otherwise, compute the homology groups of X and of Y.

Exercise 1.2. [Harder.] Repeat Exercise 1.1 with $X = S^2 \times S^2$ and $Y = \mathbb{CP}^2 \# \overline{\mathbb{CP}}^2$. Here # is the oriented connect sum operation.

Exercise 1.3. For each of the following chain complexes C_*: decide if it is exact, compute the homology groups $H_*(C)$, and compute the cohomology groups $H^*(C;\mathbb{Z})$. If it is short exact, decide if it splits.

1. $0 \to \mathbb{Z} \to 0$
2. $0 \to \mathbb{Z} \xrightarrow{1} \mathbb{Z} \to 0$
3. $0 \to \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{1} \mathbb{Z}/2\mathbb{Z} \to 0$
4. $0 \to \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{0} \mathbb{Z}/2\mathbb{Z} \to 0$
5. $0 \to \mathbb{Z} \xrightarrow{u} \mathbb{Z}^2 \xrightarrow{v} \mathbb{Z} \to 0$ — where u is the column vector $\binom{p}{q}$, where v is the row vector $(q, -p)$, and where $\gcd(p, q) = 1$.

For the next two problems we fix an abelian group G and we define $A^* = \mathrm{Hom}(A, G)$.

Exercise 1.4. Suppose that $A \to B \to C \to 0$ is an exact sequence of abelian groups. Prove that $A^* \leftarrow B^* \leftarrow C^* \leftarrow 0$ is also exact. [Thus we say that the functor $\mathrm{Hom}(\cdot, G)$ is *right exact*.]

Exercise 1.5. Suppose that $0 \to A \to B \to C \to 0$ is a split short exact sequence of abelian groups. Prove that $0 \leftarrow A^* \leftarrow B^* \leftarrow C^* \leftarrow 0$ is again a split short exact sequence.