Please let me know if any of the problems are unclear, have typos, or have any other mistakes.

Exercise 4.1. Let $\mathbb{H}$ be the quaternions. Compute the homology groups of $\mathbb{HP}^n$.

Exercise 4.2. Suppose that S is a connected surface with non-empty (topological) boundary. Prove that there is a graph $\Gamma \subset S$ so that S deformation retracts to Γ. (The following fact may be useful – every surface can be triangulated.)

Exercise 4.3. [Medium.] Suppose that S is a connected surface without boundary. Does S deformation retract to a graph?

Exercise 4.4. Give a direct proof that N_3 (the connect sum of three copies of $P^2 = \mathbb{RP}^2$) is homeomorphic to $T^2 \# P^2$ (the connect sum of the two-torus and the real projective plane).

Exercise 4.5. Recall from the classification of surfaces that

$$S_{g,b,c} \cong S^2 \# (\#_g T^2) \# (\#_b D^2) \# (\#_c P^2)$$

is a surface with g *handles*, with b *boundary components*, and with c *cross-caps*. Prove that $\chi(S_{g,b,c}) = 2 - 2g - b - c$.

Exercise 4.6. Prove that a compact, connected surface S is determined (up to homeomorphism) by its

- orientability,
- number of boundary components, and
- Euler characteristic.