

Please let me know if any of the problems are unclear, have typos, or have any other mistakes.

Exercise 5.1. Give Δ -complex structures for the following surfaces: S^2 (the two-sphere), P^2 (the real projective plane), T^2 (the two-torus), and K^2 (the Klein bottle). (Finding the smallest possible structure in each case will make the following computations easier.)

Exercise 5.2. For each surface F in Exercise 5.1 use its Δ -complex structure, and the Seifert–van Kampen theorem, to compute the fundamental group $\pi_1(F)$.

Exercise 5.3. For each surface F in Exercise 5.1 give the simplicial chain complex associated to its Δ -complex structure. Compute the resulting homology groups $H_k(F)$ as well the Euler characteristic $\chi(F)$.

Exercise 5.4. For each surface F in Exercise 5.1 give the simplicial cochain complex associated to its Δ -complex structure, using $R = \mathbb{Z}$ as the coefficient ring. Compute the resulting cohomology groups $H^k(F, \mathbb{Z})$.

Exercise 5.5. For each surface F in Exercise 5.1 compute the cup product structure (and thus the ring structure) on $H^*(F; \mathbb{Z})$.