Please let me know if any of the problems are unclear, have typos, or have any other mistakes. Please let me know if any of the problems are unclear, have typos, or have mistakes. Please turn in your solution to Exercise 4.5 on Friday (2020-02-28) before noon.

Exercise 4.1. Give Δ–complex structures for the following surfaces: S^2 (the two-sphere), P^2 (the real projective plane), T^2 (the two-torus), and K^2 (the Klein bottle). (Finding the smallest possible structure in each case will make the following computations easier.)

Exercise 4.2. For each surface F in Exercise 4.1 use its Δ–complex structure, and the Seifert–van Kampen theorem, to compute the fundamental group $\pi_1(F)$.

Exercise 4.3. Suppose that $R = \mathbb{Z}$ is the coefficient ring. For each surface F in Exercise 4.1 give the simplical chain complex associated to its Δ–complex structure. Compute the resulting homology groups $H_k(F)$ as well the Euler characteristic $\chi(F)$.

Exercise 4.4. For each surface F in Exercise 4.1 give the simplical cochain complex associated to its Δ–complex structure, using $Q = R = \mathbb{Z}$ as the coefficient module. Compute the resulting cohomology groups $H^k(F; \mathbb{Z})$.

Exercise 4.5. For each surface F in Exercise 4.1 compute the cup product structure (and thus the $\mathbb{Z}$–algebra structure) on $H^*(F; \mathbb{Z})$.