

Please let me know if any of the problems are unclear or have typos. Please let me know if you have suggestions for exercises.

Exercise 6.1. Suppose that P is a planar surface, properly embedded in the three-ball B^3 . Suppose that P has at least two boundary components. Show that there is a disk D , embedded in the interior of B^3 , so that

- $D \cap P = \partial D$ and
- ∂D separates ∂P in P .

Exercise 6.2. Suppose that T is a model tetrahedron.

- A simple closed curve $\alpha \subset \partial T$ is *normal* if it is transverse to $T^{(1)}$ and its intersection with any face is a collection of normal arcs.
- A normal curve α *doubles back* if there is an arc β , strictly contained in an edge of $T^{(1)}$, so that $\beta \cap \alpha = \partial\beta$.
- The *length* of a normal curve α is $|\alpha \cap T^{(1)}|$.

Show that a normal curve $\alpha \subset \partial T$ doubles back if and only if it has length at least five.

Exercise 6.3. Set $X = S^2 \times S^2$. Prove that X is irreducible (in the sense that any locally flat three-sphere in X bounds a homotopy four-ball). Note that $\pi_3(X) \cong \mathbb{Z}^2$. Deduce that the “obvious” generalisation of the sphere theorem to dimension four does not hold.