Please let me know if any of the problems are unclear or have typos. Also, please let me know if you have suggestions for exercises. For some of the problems I have given a (very vague) level of difficulty.

For all of the problems we use the following notation. Suppose that K is a knot in the three-sphere. Let $N(K)$ be a small closed product neighbourhood of K (Thus $N(K)$ is homeomorphic to a solid torus $S^1 \times D^2$.) Let $n(K)$ be the interior of $N(K)$. We define $X_K = S^3 - n(K)$ to be the *knot complement* for K.

Exercise 10.1. Let $K \subset S^3$ be the figure-eight knot. Let $T = \partial X_K$.

- Show that T is essential: that is, π_1–injective.
- Show that X_K is geometrically atoroidal.

Exercise 10.2. Suppose that L and L' are knots, in the three-sphere, distinct from the unknot. Let $K = L \# L'$ be their connect sum. Show that X_K is toroidal.

Exercise 10.3. Suppose that K is a knot in the three-sphere, distinct from the unknot. Show that K is a torus knot if and only if X_K is

- geometrically atoroidal but
- cylindrical

Exercise 10.4. [Hard.] Suppose that K is a knot in the three-sphere, distinct from the unknot. Show that K is a torus knot if and only if $\pi_1(X_K)$ has non-trivial centre.