Please let me (Saul) know if any of the problems are unclear or have typos. Please turn in a solution to Exercise 3.3 on the Moodle page before 2024-10-18, noon. If you collaborate with other students, or use AI, please include their names.

For the first three problems the paths $f, g, h \colon I \to X$ are loops based at the point $x_0 \in X$. The path $e \colon I \to X$ is the constant loop, also based at x_0.

Exercise 3.1. Give an explicit parameterization of the loop $e * f$. Show, by giving a picture in $I \times I$, a picture in X, and an explicit homotopy, that $e * f$ is homotopic (preserving endpoints) to f.

Exercise 3.2. Define $\bar{f} \colon I \to X$ by $\bar{f}(s) = f(1 - s)$. Give an explicit parameterization of the loop $f * \bar{f}$. Show, by giving a picture in $I \times I$, a picture in X, and an explicit homotopy, that $f * \bar{f}$ and e are homotopic (preserving endpoints). Briefly discuss the corresponding situation for $\bar{f} * f$.

Exercise 3.3. Give explicit parameterizations of the loops $p = (f * g) * h$ and $q = f * (g * h)$. Show, by giving a picture in $I \times I$, a picture in X, and an explicit homotopy, that p and q are homotopic (preserving endpoints).

Exercise 3.4.

- Let $X \subset \mathbb{R}^3$ be the union of the coordinate axes. Show that $\mathbb{R}^3 - X$ is homotopy equivalent to a graph. Which graph?
- Let $X \subset \mathbb{R}^4$ be the union of the xy–plane and the zw–plane. Show that $\mathbb{R}^4 - X$ is homotopy equivalent to a surface. Which surface?

Exercise 3.5. Suppose that $p \colon Y \to X$ is a covering map. Recall the definition of $\mathrm{Deck}(p)$ and prove it is a group (using composition of functions as the binary operation).

Exercise 3.6. Show that the map $p \colon \mathbb{R} \to S^1$ defined by $p(t) = \exp(2\pi i t)$ is a covering map. Give an informal proof that $\mathrm{Deck}(p) \cong \mathbb{Z}$. (We will give a careful proof of this later in the course.)