Please let me (Saul) know if any of the problems are unclear or have typos. Please turn in a solution to Exercise 5.3 by 12noon, on 2024-11-01, via the Moodle portal. If you collaborate with other students or AI, please include their names.

Exercise 5.1. Define the punctured plane to be $\mathbb{C}^\times = \mathbb{C} - \{0\}$. Show that the map $p\colon \mathbb{C}^\times \to \mathbb{C}^\times$ defined by $p(z) = z^2$ is a covering map. Explain why the squaring map on $\mathbb{C}$ itself is not a covering map.

Exercise 5.2. [Exercise 12, page 39, of Hatcher.] Show that for every homomorphism $\phi\colon \pi_1(S^1, 1) \to \pi_1(S^1, 1)$ there is a pointed map $f\colon (S^1, 1) \to (S^1, 1)$ so that $\phi = f_*$. In other words, f induces ϕ.

Exercise 5.3. Suppose that $p\colon \widetilde{X} \to X$ is a covering map, and suppose that $\widetilde{X}$ is path-connected. Show that $\tau \in \mathrm{Deck}(p)$ fixes a point of $\widetilde{X}$ if and only if $\tau = \mathrm{Id}_{\widetilde{X}}$.

Exercise 5.4. Suppose that $p\colon \mathbb{R} \to S^1$ is the usual covering map, namely $p(t) = \exp(2\pi i t)$. Give a complete proof that $\mathrm{Deck}(p) \cong \mathbb{Z}$.

Exercise 5.5. [Exercise 16, page 39, of Hatcher.] Show that there is no retraction $r\colon X \to A$ in any of the following cases. (Give short justifications of any fundamental group computations.)

- $X = \mathbb{R}^3$ with A any subspace homeomorphic to S^1.
- $X = S^1 \times D^2$ with A its boundary torus $S^1 \times S^1$.
- $X = S^1 \times D^2$ and A the circle shown in the figure. [See book.]
- $X = D^2 \vee D^2$ with A its boundary $S^1 \vee S^1$.
- X a disk with two points on its boundary identified and A its boundary $S^1 \vee S^1$.
- X the Möbius band and A its boundary circle.

Exercise 5.6. [Hard.] We say that a space X has the *fixed point property* if every map $f\colon X \to X$ has a fixed point. Define the *tripod* to be the set

$$T = \big\{ r\exp(2\pi i k/3) \in \mathbb{C} \mid r \in [0,1], k \in \{0,1,2\} \big\}.$$

So T is a connected graph with three edges and four vertices. Prove the tripod has the fixed point property.