Please let me (Saul) know if any of the problems are unclear or have typos. There are no problems to turn in from this exercise sheet.

Exercise 10.1. [Delzant.] Suppose that (G, S) is δ–hyperbolic. Suppose that $g \in G$ is not torsion. Let $V_{2\delta}$ be the number of points in $B_S(1_G, 2\delta)$. Prove that there are at most $V_{2\delta}$ short-lex geodesic lines in $\Gamma(G, S)$ which are bounded Hausdorff distance from the subgroup $\langle g \rangle$; furthermore, any two such intersect if and only if they are equal.

Exercise 10.2. Suppose that G is a group. Suppose that g is an element of G. Define

$$E(g) = \{h \in G \mid hg^n h^{-1} = g^{\pm n} \text{ for some } n \neq 0\}$$
$$E^+(g) = \{h \in G \mid hg^n h^{-1} = g^n \text{ for some } n \neq 0\}$$

Prove the following.

1. $E(g)$ is a subgroup of G.
2. $E^+(g)$ is an index two or one subgroup of $E(g)$.
3. $E(g^k) = E(g)$ for any $k \neq 0$.
4. $\langle g \rangle < C_G(g) < E^+(g) < E(g)$.

Exercise 10.3. [Non-examinable.] Suppose that (G, S) is a δ–hyperbolic group.

1. Look up and understand a definition of $\partial_\infty G$, the *Gromov boundary* of G.
2. Prove that $\partial_\infty F(a, b)$ is homeomorphic to a Cantor set.
3. Suppose that $F = F_g$ is the closed, connected, oriented surface of genus g. Prove that $\partial_\infty \pi_1(F)$ is homeomorphic to a circle.
4. Prove that quasi-isometries of hyperbolic groups induce homeomorphisms of their boundaries.