Please post on the Q&A forum if any of the problems have mistakes, or are unclear, or can be improved. Thank you!

Exercise 6.1. Prove the three easy directions of the removable singularity theorem. ◇

Exercise 6.2. Suppose that $f(z) = z/(z-1)$. For each choice of point z_0, find the Laurent series expansion of f in a punctured neighbourhood of z_0.

a) $z_0 = 0$ b) $z_0 = 1$ c) $z_0 = \infty$

For the last we are asking for the series in a punctured neighbourhood of infinity. ◇

Exercise 6.3. Suppose that $f(z) = 1/(z^2 - 1)$. Find the Laurent series expansion of f in a punctured neighbourhood of the point $z_0 = 1$.

What happens if you replace a punctured neighbourhood of $z_0 = 1$ with an annulus centred at a_0? As always, f must be holomorphic in the annulus. [There is now more than one solution.] ◇

Exercise 6.4. Suppose that $f(z) = \frac{1}{z-1} + \frac{1}{z-2}$. For each choice of annulus A, find the Laurent series expansion of f inside of A.

a) $A = A(0; 0, 1)$ b) $A = A(0; 1, 2)$ c) $A = A(0; 2, \infty)$

Note that the last annulus is a punctured neighbourhood of infinity. ◇